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Journal of Financial Markets 1 (1998) 221–252

Journal of
**FINANCIAL
MARKETS**

Information acquisition, information release and trading dynamics

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Abstract

The purpose of this paper is to examine how uncertainty about the timing of the public release of information affects strategic traders' incentives to acquire and trade on long- and short-lived private information. We show that this uncertainty generally induces strategic traders to trade on private information more aggressively and to acquire long-lived private information first. We also show that (1) an increase in the correlation between short- and long-lived information reduces the incentives to acquire either, and (2) when a public information release is anticipated, trading is more aggressive, prices are more informative and bid/ask spreads are smaller. Further, our analysis suggests that the design of a firm's public information systems affects the acquisition of private information and the efficiency of prices by determining what is short- and long-lived information and how they are correlated. © 1998 Elsevier Science B.V. All rights reserved.

JEL classification: D82; G14

Keywords: Strategic trading; Information acquisition; Long- and short-lived private information; Public information releases

The purpose of this paper is to examine how uncertainty about the timing of the public release of information affects investors' decisions to acquire and trade on private information about the value of a firm. We construct a model of a stock exchange, based on Kyle (1985), with specialist/dealers who set prices

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and traders who trade strategically on their private information. We introduce two types of private information: one that is publicly released with positive probability in any trading period and one that is not publicly released during trading. The former (which we will refer to as short-lived information) represents the type of private information that is either released on a regularly scheduled basis or when a significant event occurs.¹ The latter (which we will refer to as long-lived information) represents the type of private information that is normally revealed through the passage of time.²

In our model, investors with private information (referred to as strategic traders) can trade repeatedly on their information and form trading strategies. However, because some of their private information is probabilistically released in any given period, these investors are uncertain about the number of trades they can make prior to losing part of their information advantage. We show that, in each period, specialist/dealers set price equal to their expectation of the terminal value of the asset given all of their information (prior order imbalances, any publicly released information and the current order imbalance). In the linear equilibrium in our model, prior order imbalances and any publicly released information are captured by the intercept of the price function, and the current order imbalance is captured by the slope of the price function. The strategic traders correctly infer the specialist/dealers' price functions and balance profits from additional trades in a period with the induced change in the price at which they may trade both in the current and all future periods. The result of this balancing of the benefits and costs of more aggressive trading leads strategic traders to submit larger orders when there is uncertainty about the time of the release of their short-lived information.

Our analysis has a number of implications, some of which are testable.³ For example, our analysis of incentives to acquire private information suggests that

¹ In our model, the probability that this information is released in any period is exogenous, allowing us to examine different types of information releases. For example, if one wants to think of short-lived information as being similar to private information about a regularly scheduled earnings or dividend announcement, then in our model, the probability of early release is low and the probability of release around the regularly scheduled time is high. If one prefers to think of it as private information about a unusual, significant event (typically reported in an 8-K filing), then the probability of release is uniformly high if the market widely anticipates the event (examples include *widely expected* announcements of stock splits, stock repurchases, mergers and acquisitions, or changes in upper management). If the market does not anticipate the significant event, then the probability of release of this type of information is uniformly low, (examples include *unexpected* announcements of management forecasts of earnings, the death of a CEO, the filing of a significant lawsuit, or any other significant event).

² Examples include the outcome of strategic plans or long-term projects.

³ Because it is difficult to examine private information issues when using data generated in naturally occurring markets, some of these implications are well-suited for examination in laboratory asset markets.

if the costs of acquiring short- and long-lived information are approximately the same, then:

- If the variance of the long-lived information exceeds the variance of the short-lived information and strategic traders acquire only one, they choose to acquire the long-lived information. (They can trade on it longer and the profits earned in each period are greater.)
- As the correlation between the short- and long-lived information increases, the profits from acquiring long-lived information decline and may result in neither type of information being acquired. (If the short-lived information is highly correlated with the long-lived information, then once the short-lived information is released, trading on the long-lived information is not very profitable.)
- An increase in the probability that the short-lived information will be released soon may result in neither type of information being acquired, given that the short- and long-lived information are positively correlated. (A more probable release of the short-lived information reduces the expected profits from trading on both types of information.)

Our analysis also has implications for the design of firms' public information systems. Such systems (e.g., accounting reports, SEC filings, press releases, conferences with analysts, and any other means by which the firm provides information to the market) determine what short- and long-lived information represent by determining what is released (and when) to the public.⁴ The content of a firm's public information releases also determines the degree to which the short- and long-lived information are correlated as well as their prior variances. As we noted above, private information acquisition depends on the characteristics of short- and long-lived information and is thus dependent upon the firm's public information systems. Our analysis suggests the following:

- If the public information systems ensure that the short- and long-lived information are uncorrelated, it is optimal to release short-lived information rapidly. This is because rapid release reduces exogenous traders' losses by equalizing information access and allows the information to be impounded in price quickly without reducing private incentives to learn the long-lived information.
- However, if the public information systems ensure that the short- and long-lived information are positively correlated, rapid release of short-lived information reduces (and may totally eliminate) private incentives to learn both

⁴ For an alternative view of a firm's public information systems, see Baiman and Verrecchia (1996).

short- and long-lived information. Hence, fewer traders acquire long-lived information and so it is impounded in price more slowly, if at all. This raises the possibility of misdirected investment and social loss.

Finally, our analysis offers insight into the effects of announcing that there will be a public information release in the near future.⁵ In our model, the announcement that there will be a release affects the way strategic traders profit on their private information. In particular, such an announcement causes them to hurry to exploit their short-lived private information by trading more aggressively. This results in more informative prices and smaller effective bid/ask spreads in front of the release. Hence, our analysis suggests that there are benefits to encouraging firms to reduce uncertainty about the timing of their public information releases.

Our paper contributes to the literature in accounting and finance that examines the effect of a public release of information. Most of this literature employs a rational expectations framework and has, as the central focus, comparative static analysis of changes in characteristics of the public signal or private information on the variance of the price change around a release or on expected trading volume.⁶ Recently, Holden and Subrahmanyam (1996) extended this literature by studying traders who become privately informed by *either* acquiring information that is released early or (independent) information that is released later. Their numerical simulations suggest that traders' risk tolerance determines whether all choose to privately acquire information that is released early, all choose to privately acquire information that is released later, or some choose to privately acquire one and some choose to privately acquire the other information.⁷ Thus, our work is closely related to theirs. However, our structure differs in four important ways: (1) ours is a dynamic model in that traders submit multiple orders without new information; (2) ours is a strategic model in that traders understand that their order affects the price at which trades are executed;

⁵ Examples include firms calling a press conference, scheduling meetings with analysts or announcing that they will be releasing earnings on a particular day.

⁶ See Demski and Feltham (1994), Kim and Verrecchia (1994), McNichols and Trueman (1994), Kim and Verrecchia (1991a,b) and Holthausen and Verrecchia (1988). Grundy and McNichols (1989) show that a public signal may partially substitute for information acquired through additional trading.

⁷ Lundholm (1991) presents a static rational expectations model in which heterogeneously informed traders have access to two 'true value plus noise' private signals. The noise in the signals is potentially correlated. There is also a 'true value plus noise' public signal whose noise is uncorrelated with the noise in the private information. He shows that for certain parameter values, rational expectations equilibria exist in which each trader acquires neither or both pieces of private information and other parameters for which some traders acquire one piece of information and the rest acquire the other. The public signal introduces a tendency for traders to choose to either acquire both pieces of private information or neither.

(3) there is uncertainty about the time of the release of part of the strategic traders' private information and (4) strategic traders are symmetrically informed.

The paper is organized as follows. The next section contains a description of the model and the derivation of equilibrium. In Section 2, we derive implications from our analysis and we conclude in Section 3. All proofs are in the appendix.

1. The model

In our model, an asset is traded by three types of agents: strategic traders, exogenous traders and specialist/dealers. Strategic traders trade on private information; exogenous traders trade for unmodelled reasons; and specialist/dealers execute trades. The terminal value of the risky asset, v , is the sum of three normally distributed random variables, r , η and ε . We interpret r as private information known to strategic traders and available for public release during trading; η as additional private information known to strategic traders but not available for public release during trading; and ε as noise. We assume that $E[\varepsilon] = 0$, ε is independent of all other random variables and r and η are either uncorrelated or positively correlated.⁸

Trade takes place in a stylized model of a stock market based on Kyle (1985). In this model, both strategic and exogenous traders submit market orders⁹ which are aggregated yielding the period's net order imbalance. Specialist/dealers are charged with setting a price after observing the imbalance and then mopping up by trading on their own accounts. Trades are executed at the best quoted price for traders. If there is a buy imbalance, trades are executed at the lowest price quoted by specialist/dealers and if there is a sell imbalance, at the highest price quoted. Trade takes place in each of T periods and, in each period there is a probability, q_t , that r is publicly released.¹⁰ Because exogenous

⁸ This information structure means that strategic traders know for certain what information can be publicly released. One can readily relax this by assumption without altering our results.

⁹ A positive order is a request to buy shares and a negative order a request to sell shares.

¹⁰ In this paper information is publicly released between trading periods. Such a model is a good description of information releases in call markets and so we can use our model to study releases made either while the NYSE is closed or while trading is halted. We could have assumed that r is announced during the period, after traders submit their orders but before specialist/dealers price and execute them. Such a model is a good description of information releases in continuous markets. Since this is the trading mechanism in use during the trading day on the NYSE, we could use this model to study releases made during trading. We believe that an analysis of this second model would yield results that are virtually identical to ours. Basically, there is only one difference between the models. In the second, strategic traders submit orders which are larger than they would have been had r been released prior to their order submission but smaller than they would have been had r not been released until after their orders were executed. Specialist/dealers have the same information and so choose to execute the imbalance using a price function analogous to the one in our model.

traders trade for unmodelled reasons, we take their aggregate trade in period t to be a normally distributed random variable u_t , with mean zero and variance σ_u^2 . We assume that u_t is independent of all other random variables (including the exogenous traders' aggregate trade in other periods). Our N strategic traders are risk neutral profit maximizers and are endowed with their private information.¹¹

Strategic traders may trade repeatedly on their private information and so we must solve for their optimal dynamic trading strategy. This means that we seek a perfect Bayes Nash equilibrium to our game.¹² The solution concept requires that current actions be optimal given what has transpired. We are interested in the effect of uncertainty in the time of public releases of information. This uncertainty means that trading strategies depend on optimal actions conditional on r having been released and optimal actions conditional on it not yet having been released. As a result, the building blocks for our general model are the equilibrium trading strategies and price functions when r is not released and when it is released prior to the start of trading. Thus, we first consider the case in which r is not publicly released during trading. Then, we consider the case in which r is publicly released prior to *any* trading. Finally, we consider the general case in which r is released in period t with probability q_t .

1.1. Equilibrium when r is not publicly released during trading

1.1.1. Specialist/dealers' decision problems

In this special case, the probabilities of release are all zero ($q \equiv 0$); the $\mathcal{K} \geq 2$ specialist/dealers have the same prior belief about the asset's value, $E[v]$; and strategic traders have two sources of private information, r and η . In period t , specialist/dealers have observed price and order imbalance in all previous periods and observe the current period's order imbalance. Let the order imbalance in t be z_t , the sum of the orders placed by strategic and exogenous traders ($\sum_{j=1}^N x_{j,t} + u_t$), and let $\mathbf{z}_t \equiv (z_1, z_2, \dots, z_t)$. Each specialist/dealer is a risk neutral expected profit maximizer who, in each period, chooses a price at which she is willing to execute all orders. The orders are then executed at the price most favorable for the traders. Thus, a specialist/dealer's expected profits in t are

$$E \left[(p_t(z_t; \mathbf{0}) - v) \frac{z_t}{K_t} \middle| \mathbf{z}_t, \mathbf{p}_{t-1} \right]$$

¹¹ Since there are a finite number of trading periods, the analysis will be done without discounting. Introducing discounting has no qualitative effects but adds to the notational complexity.

¹² We must determine optimal trades for strategic traders and optimal prices for specialist/dealers in each period given what has transpired and given their beliefs. If possible, the beliefs must satisfy Bayes' rule. Since all random variables are normally distributed, we can use Bayes' rule in all situations. See Gibbons (1993) or Fudenberg and Tirole (1993) for additional details.

where K_t is the number of specialist/dealers who quote the best price and $\mathbf{p}_{t-1}$ is the vector of execution prices in previous periods.¹³

Since specialist/dealers choose prices at which they are willing to execute the imbalance and since it is executed at the best price, Bertrand competition results in prices being driven down until each specialist/dealer earns zero expected profits in equilibrium. In fact, in equilibrium, each earns zero expected profits in each period.¹⁴ Thus, specialist/dealers quote the same price, found by setting expected profits to zero in each period,

$$p_t(\mathbf{z}_t; \mathbf{0}) = E[v|\mathbf{z}_t, \mathbf{p}_{t-1}, \mathbf{0}].$$

In other words, in equilibrium, risk neutral specialist/dealers set price equal to their expectation of the asset's value given all of their information.

We restrict attention to linear equilibria which means that we seek equilibrium price functions which are linear in the order imbalance. As a result, we conjecture that

$$p_t(\mathbf{z}_t; \mathbf{0}) = k_t(\mathbf{0}) + \sum_{\tau=1}^{t-1} b_\tau^t(\mathbf{0})z_\tau + \lambda_t(\mathbf{0})z_t = \mu_t(\mathbf{0}) + \lambda_t(\mathbf{0})z_t \quad t = 1, 2, \dots, T.$$

Since the execution price is a known function of the vector of order imbalances, there is no additional information in prices and so $E[v|\mathbf{z}_t, \mathbf{p}_{t-1}, \mathbf{0}] = E[v|\mathbf{z}_t, \mathbf{0}] = E[v|\mathbf{p}_{t-1}, z_t, \mathbf{0}]$.

1.1.2. Strategic traders' decision problems and equilibrium

To complete the analysis when r is not publicly released during trading ($\mathbf{q} \equiv \mathbf{0}$), we solve for strategic traders' equilibrium trading strategies assuming that the equilibrium price function is linear and then show that if specialist/dealers correctly infer equilibrium trading strategies, their equilibrium strategies result in a linear equilibrium. As usual, this is done by analyzing the last period first and working backward.

There are N risk neutral, expected profit maximizing strategic traders who are endowed with the same private information, r and η . In the last period, T , the i th

¹³ The notation implicitly assumes that if more than one specialist/dealer quotes the best price, they split the order imbalance equally. This is the standard assumption for Bertrand competitors but it is well known that the results do not depend on the particular allocation of the order imbalance. Further, because specialist/dealers are risk neutral, the analysis would not be affected if a random allocation of the order imbalance were employed.

¹⁴ Were specialist/dealers' expected profits zero but not zero in each period, they would earn positive expected profits in some periods and negative expected profits in others. This cannot be in equilibrium because specialist/dealers can avoid the negative profits by setting a price that ensures that they execute no shares.

strategic trader chooses the number of shares to trade, $x_{i,T}$, to maximize $E[(v - p_T(z_T; \mathbf{0}))x_{i,T} | z_{T-1}, r, \eta, \mathbf{0}]$. The first order condition yields¹⁵

$$x_{i,T} = \frac{1}{2\lambda_T(\mathbf{0})} \left(r + \eta - \mu_T(\mathbf{0}) - \lambda_T(\mathbf{0}) \sum_{j \neq i}^N x_{j,T} \right).$$

Solving in the usual way shows that each strategic trader uses the same strategy¹⁶

$$x_T(r, \eta, \mathbf{0}) = \frac{\alpha_T(\mathbf{0})}{N} (r + \eta - \mu_T(\mathbf{0}))$$

where $\alpha_T(\mathbf{0}) \equiv N/(N+1)\lambda_T(\mathbf{0})$. Strategic traders buy shares if they believe that the value of the asset exceeds the intercept of the equilibrium price function.

To complete the analysis of period T , we solve for $\mu_T(\mathbf{0})$ and $\lambda_T(\mathbf{0})$. To solve for $\mu_T(\mathbf{0})$, recall that specialist/dealers set $p_T(z_T; \mathbf{0}) = E[v | z_T, \mathbf{0}]$. Hence,

$$p_T(z_T; \mathbf{0}) = E[v | z_T, \mathbf{0}] = \mu_T(\mathbf{0}) + \lambda_T(\mathbf{0})z_T. \quad (1)$$

Taking expectations with respect to z_T conditional on z_{T-1} , $E[v | z_{T-1}, \mathbf{0}] = \mu_T(\mathbf{0}) + \lambda_T(\mathbf{0})E[z_T | z_{T-1}, \mathbf{0}]$. Since $z_T = Nx_T(r, \eta, \mathbf{0}) + u_T$, u_T is independent of all random variables and $E[u_T] = \mathbf{0}$, $E[z_T | z_{T-1}, \mathbf{0}] = NE[x_T(r, \eta, \mathbf{0}) | z_{T-1}, \mathbf{0}]$. Substituting for $x_T(r, \eta, \mathbf{0})$, $E[x_T(r, \eta, \mathbf{0}) | z_{T-1}, \mathbf{0}] = (\alpha_T(\mathbf{0})/N)(E[v | z_{T-1}, \mathbf{0}] - \mu_T(\mathbf{0}))$ and so $E[v | z_{T-1}, \mathbf{0}] = \mu_T^*(\mathbf{0})$. Specialist/dealers set the intercept of the equilibrium price function equal to their expectation of the value of the asset given all information prior to period T making it equal to the price at which trades were executed in $T-1$.

To compute $\lambda_T(\mathbf{0})$, multiply through Eq. (1) by z_T and take expectations with respect to z_T conditional on z_{T-1} . Doing so and rearranging yields

$$\lambda_T(\mathbf{0}) = \frac{E[vz_T | z_{T-1}, \mathbf{0}]}{E[z_T^2 | z_{T-1}, \mathbf{0}]} \equiv \frac{\text{Cov}[v, z_T | z_{T-1}, \mathbf{0}]}{\text{Var}[z_T | z_{T-1}, \mathbf{0}]}.$$

Substituting for z_T , $\lambda_T^*(\mathbf{0})$ is implicitly defined by¹⁷

$$\lambda_T(\mathbf{0}) = \left(\frac{N\alpha_T(\mathbf{0})}{\text{Var}[u_T]} \right) \text{Var}[r + \eta | z_{T-1}].$$

Hence, the slope of the equilibrium price function is ‘essentially’ proportional to the specialist/dealers’ residual variance.

¹⁵ The second order condition requires that the slope of the equilibrium price function be positive.

¹⁶ This is solved just like a Cournot game by summing the first order conditions over strategic traders and then substituting for the sum of the orders placed by other strategic traders in the first order condition.

¹⁷ This yields a quadratic in $\lambda_T^*(\mathbf{0})$ which has one positive root.

Backing up to the strategic traders' $T - 1$ period problem, each maximizes

$$E[(v - p_{T-1}(z_{T-1}; \mathbf{0})x_{i,T-1}(\mathbf{0}) + V_T(\mathbf{0})|z_{T-2}, r, \eta, \mathbf{0}]$$

where $V_T(\mathbf{0})$ is the value of the strategic trader's expected future profits from trading in T when he uses $x_T(r, \eta, \mathbf{0})$. By direct computation,

$$V_T(\mathbf{0}) = \frac{\alpha_T(\mathbf{0})(1 - \lambda_T(\mathbf{0})\alpha_T(\mathbf{0}))}{N}(r + \eta - \mu_T^*(\mathbf{0}))^2.$$

What is unknown in this expression is $\mu_T^*(\mathbf{0})$ because z_{T-1} is unknown in $T - 1$.¹⁸ Thus each maximizes expected profits from his current trade plus his expected profits on all future trades. The important detail is that buying an additional share in $T - 1$ generates expected profits in this period but is also expected to increase the price of every share purchased in the current and subsequent periods.

Using $\mu_T^*(\mathbf{0}) = E[v|z_{T-1}, \mathbf{0}] = \mu_{T-1}(\mathbf{0}) + \lambda_{T-1}(\mathbf{0})z_{T-1}$, the i th strategic trader maximizes

$$\begin{aligned} & \left(r + \eta - \mu_{T-1}(\mathbf{0}) - \lambda_{T-1}(\mathbf{0}) \sum_{j=1}^N x_{j,T-1} \right) x_{i,T-1} + \phi_{T-1}(\mathbf{0}) \\ & \left[(r + \eta - \mu_{T-1}(\mathbf{0}))^2 - 2(r + \eta - \mu_{T-1}(\mathbf{0}))\lambda_{T-1}(\mathbf{0}) \sum_{j=1}^N x_{j,T-1} + \lambda_{T-1}(\mathbf{0})^2 \right. \\ & \left. \times \left(\left(\sum_{j=1}^N x_{j,T-1} \right)^2 + \text{Var}[u_{T-1}] \right) \right], \end{aligned}$$

where $\phi_{T-1}(\mathbf{0}) \equiv \gamma_T(\mathbf{0})$ and $\gamma_T(\mathbf{0}) \equiv \alpha_T(\mathbf{0})(1 - \lambda_T^*(\mathbf{0})\alpha_T(\mathbf{0}))/N$. Following the approach used above, the first order conditions can be used to solve for each strategic trader's equilibrium trade

$$x_{T-1}(r, \eta, \mathbf{0}) = \frac{\alpha_{T-1}(\mathbf{0})}{N}(r + \eta - \mu_{T-1}(\mathbf{0})),$$

where

$$\begin{aligned} \alpha_{T-1}(\mathbf{0}) & \equiv \frac{N(1 - 2\phi_{T-1}(\mathbf{0})\lambda_{T-1}(\mathbf{0}))}{\lambda_{T-1}(\mathbf{0})(1 + N(1 - 2\phi_{T-1}(\mathbf{0})\lambda_{T-1}(\mathbf{0})))} \\ & \equiv \frac{NA_{T-1}(\mathbf{0})}{\lambda_{T-1}(\mathbf{0})(1 + NA_{T-1}(\mathbf{0}))}. \end{aligned}$$

¹⁸ The number of strategic traders is known to all. Also, $\lambda_T^*(\mathbf{0})$ is independent of the realized order imbalance in $T - 1$ because the random variables are joint normally distributed.

Further, computations analogous to those for period T yield

$$\mu_{T-1}^*(\mathbf{0}) = E[v|z_{T-2}, \mathbf{0}],$$

$$\lambda_{T-1}(\mathbf{0}) = \frac{E[vz_{T-1}|z_{T-2}, \mathbf{0}]}{E[z_{T-1}^2|z_{T-2}, \mathbf{0}]} = \left(\frac{N\alpha_{T-1}(\mathbf{0})A_{T-1}(\mathbf{0})}{\text{Var}[u_T]} \right) \text{Var}[r + \eta|z_{T-2}].$$

Let $A_t(\mathbf{0}) \equiv 1 - 2\phi_t(\mathbf{0})\lambda_t(\mathbf{0})$;

$$\alpha_t(\mathbf{0}) \equiv \frac{NA_t(\mathbf{0})}{\lambda_t(\mathbf{0})(1 + NA_t(\mathbf{0}))}; \quad \gamma_t(\mathbf{0}) = \frac{\alpha_t(\mathbf{0})(1 - \lambda_t(\mathbf{0})\alpha_t(\mathbf{0}))}{N},$$

and $\phi_t(\mathbf{0}) = (\gamma_{t+1}(\mathbf{0}) + \phi_{t+1}(\mathbf{0})(1 - \lambda_{t+1}(\mathbf{0})\alpha_{t+1}(\mathbf{0}))^2)$ with $\phi_T(\mathbf{0}) = 0$. Using these and based on the above calculations, the following proposition can be proved by induction.¹⁹

Proposition 1. For all N , if r is not released during trading, in equilibrium, specialist/dealers all choose the following price functions:

$$p_t(z_t, \mathbf{0}) = \mu_t^*(\mathbf{0}) + \lambda_t^*(\mathbf{0})z_t, \quad t = 1, 2, \dots, T$$

where $\mu_t^*(\mathbf{0}) = E[v|z_{t-1}, \mathbf{0}]$, $\lambda_t^*(\mathbf{0})$ is the solution to

$$\lambda_t(\mathbf{0}) = \left(\frac{NA_t(\mathbf{0})\alpha_t(\mathbf{0})}{\text{Var}[u_t]} \right) \text{Var}[r + \eta|z_{t-1}]$$

and strategic traders all choose the same trading strategies:

$$x_t(r, \eta, \mathbf{0}) = \frac{\alpha_t(\mathbf{0})}{N}(r + \eta - \mu_t^*(\mathbf{0})), \quad t = 1, 2, \dots, T.$$

In each period, specialist/dealers set the intercept of the current period's equilibrium price function equal to the price at which trades were executed in the previous period. In the first period, the intercept is equal to their unconditional expectation of the value of the asset. If there are at least two strategic traders, the slope of the equilibrium price function declines through time.²⁰ That is, $\lambda_t^*(\mathbf{0}) > \lambda_{t+1}^*(\mathbf{0})$. The reason later prices are less sensitive to an order imbalance is that prior order imbalances are informative. Also, the slope of the equilibrium price function is larger the more correlated are the two sources of private information. The higher the correlation, the greater the specialist/dealers' residual variance and so the more sensitive price is to an order imbalance.

¹⁹ All proofs are in the Appendix.

²⁰ See the proof of Corollary 1 in the Appendix. While we believe that this relationship holds when there is only one strategic trader, we have not yet been able to show this.

1.2. Equilibrium when r is publicly released prior to the start of trading

When r is publicly released prior to the start of trading, $q \equiv 1$. The only difference in the analysis (relative to Section 1.1) is that expectations are taken with respect to r , too, as it provides information about the terminal value of the asset.

Proposition 2. For all N , if public information r is released prior to the start of trading, in equilibrium, specialist/dealers all choose the following price functions:

$$p_t(z_t, r, \mathbf{1}) = \mu_t^*(\mathbf{1}) + \lambda_t(\mathbf{1})^* z_t, \quad t = 1, 2, \dots, T$$

where $\mu_t^*(\mathbf{1}) = E[v|z_{t-1}, r]$, $\lambda_t^*(\mathbf{1})$ is the solution to

$$\lambda_t(\mathbf{1}) = \left(\frac{N A_t(\mathbf{1}) \alpha_t(\mathbf{1})}{\text{Var}[u_t]} \right) \text{Var}[\eta|z_{t-1}, r]$$

and strategic traders all choose the same trading strategies:

$$x_t(r, \eta, \mathbf{1}) = \frac{\alpha_t(\mathbf{1})}{N} (r + \eta - \mu_t^*(\mathbf{1})), \quad t = 1, 2, \dots, T.$$

As before, specialist/dealers set the intercept equal to their expectation of the asset's value given the information they have observed prior to t (which now includes r). If there are at least two strategic traders, the slope of the equilibrium price function declines through time and the slope is larger the more correlated are the two sources of private information.²¹

Before solving the general model, we compare equilibrium outcomes when r is not released during trading to equilibrium outcomes when it is released prior to the start of trading. Beginning with equilibrium price functions, in both cases, the intercept in t equals the specialist/dealers' expectation of the value of the asset given all of the information available to them at the start of period t . Using Propositions 1 and 2,

$$\mu_t^*(\mathbf{1}) - \mu_t^*(\mathbf{0}) = (r - E[r|z_{t-1}, \mathbf{0}]) + (E[\eta|z_{t-1}, r] - E[\eta|z_{t-1}, \mathbf{0}]). \quad (2)$$

If r and η are either independent or positively correlated, then $\mu_t^*(\mathbf{1}) > \mu_t^*(\mathbf{0})$ if and only if $r > E[r|z_{t-1}, \mathbf{0}]$. If strategic traders have 'good news' about r , i.e., they know that it exceeds the market's expectation of its value, the intercept when r is released prior to trading exceeds the intercept when it is not released during trading. If strategic traders have 'bad news' about r , the intercept when r is released prior to trading is smaller than the intercept when it is not released

²¹ See the proof of Corollary 2 in the Appendix.

during trading.²² Comparing the slopes of the equilibrium price functions, prices are more sensitive to a current order imbalance if specialist/dealers do not know r — $\lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{1})$.²³ The reason is that if r is not released during trading, specialist/dealers are more uncertain about the terminal value of the asset and so make price more sensitive to an order imbalance to protect themselves when trading with the better informed traders. Further, the difference grows the greater the prior variance of r and the greater the correlation between r and η . It shrinks the greater the prior variance of η . If there is great uncertainty about r , if the report reduces the specialist/dealers' residual uncertainty (their uncertainty about η), or if there is little uncertainty about η , price adjustment to an order imbalance is smaller when r is released prior to the start of trading.

To compare the equilibrium trading strategies, we use a result obtained in the proof of Proposition 3: $A_t(\mathbf{0}) = A_t(\mathbf{1}) \equiv A_t$. Thus,

$$x_t(r, \eta, \mathbf{0}) = \frac{A_t}{1 + NA_t} \left(\frac{r + \eta - \mu_t^*(\mathbf{0})}{\lambda_t^*(\mathbf{0})} \right),$$

$$x_t(r, \eta, \mathbf{1}) = \frac{A_t}{1 + NA_t} \left(\frac{r + \eta - \mu_t^*(\mathbf{1})}{\lambda_t^*(\mathbf{1})} \right).$$

Because $\lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{1})$ and $\mu_t^*(\mathbf{1}) < \mu_t^*(\mathbf{0})$ when $r < E[r|z_t, \mathbf{0}]$ (see Eq. (2)), $x_t(r, \eta, \mathbf{0}) < x_t(r, \eta, \mathbf{1})$. In previous work using Kyle models, $r < E[r|z_{t-1}, \mathbf{0}]$ implies that strategic traders sell shares. Here, it does not. The reason is that they have two sources of private information, r and η , and they sell shares if their expectation of the value of the asset is smaller than the specialist/dealers' expectation. Since r and η are (weakly) positively correlated, if $r < E[r|z_{t-1}, \mathbf{0}]$ then it is (weakly) more likely but *not* certain that $\eta < E[\eta|z_{t-1}, r]$. Having bad news about r does not ensure that strategic traders also have bad news about η . Since strategic traders sell (buy) shares if *overall* they have bad (good) news, two sources of information permits the possibility that they sell when r is not released during trading and buy when r is released prior to the start of trading.²⁴ If $\eta < E[\eta|z_{t-1}, r]$, they sell shares whether or not r is released. We conclude that

²² The reader may wonder why the relationship between the intercepts does not depend on whether traders have good or bad news about η . The reason is that in both cases (when r is not released during trading or when it is released prior to the start of trading) η is known only by strategic traders. In either case, it is information that specialist/dealers are trying to infer.

²³ See the proof of Proposition 3 in the Appendix.

²⁴ For example, if they have bad news about r (i.e., $r < E[r|z_{t-1}, \mathbf{0}]$) and good news about η (i.e., $\eta > E[\eta|z_{t-1}, r]$), when r is released prior to the start of trading, strategic traders buy shares. When r is not released during trading, strategic traders may buy but may sell shares. Which they do depends on the relative magnitudes of $r - E[r|z_{t-1}, \mathbf{0}]$ and $\eta - E[\eta|z_{t-1}, r]$. The latter need not offset the former. If not, they sell shares when r is not released during trading and buy them if it is released prior to the start of trading.

if strategic traders have bad news, in both cases they sell shares and they sell more shares when r is released prior to trading than they do when r is not released during trading. If they have bad news about r and good news about η but overall bad news, they sell shares when r is not released during trading but buy if it is released prior to the start of trading.

1.3. Equilibrium with uncertain time of public release of r

1.3.1. Specialist/dealers' decision problems

We now consider the more general case when there is a probability, q_t , of public release of r in t . If r is released in t , it is released prior to the start of trading and so specialist/dealers either set prices knowing r or knowing that it cannot be released until the next period. Thus, the previous analysis of their decisions applies and each specialist/dealer sets her price in t equal to her expectation of the value of the asset given *all* of the information available to her at t ,

$$p_t(z_t; q) = \begin{cases} E[v | z_t, r] & \text{if } r \text{ was released,} \\ E[v | z_t, q] & \text{if not.} \end{cases}$$

1.3.2. Strategic traders' decision problems and equilibrium

In T , if r was not released, it will not be released at all and so each strategic trader uses the equilibrium trading strategy that he would use had $q = 0$, $x_T(r, \eta, 0)$. If r was released in T , each trader uses the equilibrium trading strategy that he would use had $q = 1$, $x_T(r, \eta, 1)$. The equilibrium coefficients are

$$\mu_T^*(q) = \begin{cases} \mu_T^*(1) & \text{if } r \text{ was released,} \\ \mu_T^*(0) & \text{if not,} \end{cases}$$

$$\lambda_T^*(q) = \begin{cases} \lambda_T^*(1) & \text{if } r \text{ was released,} \\ \lambda_T^*(0) & \text{if not.} \end{cases}$$

In $T - 1$, if r was released prior to trading, the analysis in Section 1.2.2 applies. If it is not yet released, trade takes place without r being public information but with a probability, q_T , that it will be released prior to trading in T . Thus, the i th strategic trader maximizes

$$E[(v - p_{T-1}(z_{T-1}; q))x_{i,T-1} + ((1 - q_T)V_T(0) + q_TV_T(1))(z_{T-2}, r, \eta, q)].$$

The first term is his expected profits from the current trade and the second term is his expected profits on future trades. The latter is a weighted sum of his expected profits from trading if r is not released and his expected profits from trading if it is. Strategic traders balance the expected profits from buying (selling) additional shares in $T - 1$ with the associated increase (decrease) in the price at which they will be able to trade shares in T . As before, the impact on the price in

T is uncertain because the order imbalance in $T - 1$ is unknown to strategic traders. There is additional uncertainty though because r may be publicly released prior to trading in T .

Substituting for $E[V_T(\mathbf{0})|z_{T-2}, r, \eta]$ and $E[V_T(\mathbf{1})|z_{T-2}, r, \eta]$, using the expressions developed in Section 1.1.2 and 1.2.2 respectively, and solving as in those sections yields the equilibrium trading strategy for each strategic trader if r was not released prior to $T - 1$,

$$x_{T-1}(r, \eta, \mathbf{q}) = \frac{\delta_{T-1}(r + \eta - \mu_{T-1}(\mathbf{q})) - d_{T-1}(r + \eta - \mu_{T-1}^*(\mathbf{1}))}{\lambda_{T-1}(\mathbf{q})[1 + N\delta_{T-1}] - \lambda_{T-1}(\mathbf{1})Nd_{T-1}},$$

where $\delta_{T-1} \equiv 1 - 2(1 - q_T)\gamma_T(\mathbf{0})\lambda_{T-1}(\mathbf{q})$, and $d_{T-1} \equiv 2q_{T\gamma T}(\mathbf{1})\lambda_{T-1}(\mathbf{1})$.

If r is released prior to the start of trading in $T - 1$, then $\mu_{T-1}^*(\mathbf{q}) = E[v|z_{T-2}, r] = \mu_{T-1}^*(\mathbf{1})$. If not, the analysis is *not* the same as the analysis when r is not released during trading because there is a chance that r will be released prior to the start of trading in T . We compute the coefficients of the equilibrium price function as before noting that $E[z_{T-1}|z_{T-2}] = 0$.

$$\mu_{T-1}^*(\mathbf{q}) = \begin{cases} E[v|z_{T-2}, r] = \mu_{T-1}^*(\mathbf{1}) & \text{if } r \text{ was released prior to } T - 1, \\ E[v|z_{T-2}] = \mu_{T-1}^*(\mathbf{0}) & \text{if not.} \end{cases}$$

Each specialist/dealer sets the intercept equal to her expectation of the value of the asset given all of her information. If r is released prior to $T - 2$, she sets the intercept equal to the price at which she executed trades in $T - 2$, $\mu_{T-1}^*(\mathbf{1})$. If r is not released prior to $T - 1$, she sets the intercept equal to the price at which she executed trades in $T - 2$, $\mu_{T-1}^*(\mathbf{0})$. If r is released prior to $T - 1$, information is released after executing trades in $T - 2$ and prior to trading in $T - 1$ and so her expectation of the value of the asset is updated to reflect her additional information, r .

We proceed as we did in the prior sections to obtain an equation that $\lambda_{T-1}^*(\mathbf{q})$ must satisfy:

$$\lambda_{T-1}(\mathbf{q}) = \begin{cases} \frac{E[vz_{T-1}|z_{T-2}, r]}{E[z_{T-1}^2|z_{T-2}, r]} & \text{if } r \text{ was released prior to } T - 1, \\ \frac{E[vz_{T-1}|z_{T-2}]}{E[z_{T-1}^2|z_{T-2}]} & \text{if not.} \end{cases}$$

If r was released, $\lambda_{T-1}^*(\mathbf{q})$ solves the same equation as $\lambda_{T-1}^*(\mathbf{1})$ and so $\lambda_{T-1}^*(\mathbf{q}) = \lambda_{T-1}^*(\mathbf{1})$. If r is not yet released, $\lambda_{T-1}^*(\mathbf{q}) \neq \lambda_{T-1}^*(\mathbf{0})$ because r might be released in the future. Intuitively, the reason that a linear equilibrium exists (that is, an equilibrium in which it is optimal for market makers to choose the same linear price function) is that the market makers' conditional expectation of profits are a linear combination of their expected profits in each of the two regimes – when the information is released and when it is not. Based on the above, the following can be proved by induction.

Theorem 1. For all N , if $q_t \in [0, 1]$ in each period t , in equilibrium, specialist/dealers all choose the following price functions:

$$p_t(z_t; \mathbf{q}) = \mu_t^*(\mathbf{q}) + \lambda_t^*(\mathbf{q})z_t, \quad t = 1, 2, \dots, T$$

where

$$\mu_t^*(\mathbf{q}) = \begin{cases} E[v|z_{t-1}, r] \equiv \mu_t^*(\mathbf{1}) & \text{if } r \text{ is released prior to } t-1, \\ E[v|z_{t-1}] \equiv \mu_t^*(\mathbf{0}) & \text{if not} \end{cases}$$

and $\lambda_t^*(\mathbf{q}) = \lambda_t^*(\mathbf{1})$ if r was released prior to t and solves

$$\lambda_t(\mathbf{q}) = \frac{E[v, z_t|z_{t-1}]}{E[z_t^2|z_{t-1}]}$$

if not. Strategic traders all choose the same trading strategies:

$$x_t(r, \eta, \mathbf{q}) = \begin{cases} x_t(r, \eta, \mathbf{1}) & \text{if } r \text{ was released prior to } t, \\ \frac{\delta_t(r + \eta - \mu_t^*(\mathbf{q})) - d_{t-1}(r + \eta - \mu_t^*(\mathbf{1}))}{\lambda_t^*(\mathbf{q})[1 + N\delta_t] - \lambda_t^*(\mathbf{1})Nd_t} & \text{if not.} \end{cases}$$

1.3.3. Comparisons

We can compare the equilibrium price functions in the three information release cases. From Theorem 2, the intercept in t equals the specialist/dealers' expectation of the value of the asset given what they know at t . Comparing slopes is facilitated by first re-expressing trading strategies,

$$\begin{aligned} x_t(r, \eta, \mathbf{q}) &= \left(\frac{N}{\lambda_{t-1}(\mathbf{q})[1 + N\delta_{t-1}] - \lambda_{t-1}(\mathbf{1})Nd_{t-1}} \right) \\ &\quad \left[\frac{\delta_t}{\alpha_t(\mathbf{0})} x_t(r, \eta, \mathbf{0}) - \frac{d_t}{\alpha_t(\mathbf{1})} x_t(r, \eta, \mathbf{1}) \right] \\ &= \frac{1}{\lambda_t^*(\mathbf{1})} [(1 - w_t)\lambda_t^*(\mathbf{0})x_t(r, \eta, \mathbf{0}) + w_t\lambda_t^*(\mathbf{1})x_t(r, \eta, \mathbf{1})] \end{aligned} \quad (3)$$

where $w_t = [-Nd_t/\alpha_t(\mathbf{1})\lambda_{t-1}(\mathbf{q})(1 + N\delta_{t-1}) - \lambda_{t-1}(\mathbf{1})Nd_{t-1}]$.²⁵ If $\lambda_t^*(\mathbf{0}) = \lambda_t^*(\mathbf{1})$, then $x_t(r, \eta, \mathbf{q})$ is a weighted average of $x_t(r, \eta, \mathbf{0})$ and $x_t(r, \eta, \mathbf{1})$. Since $\lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{1})$, the equilibrium trading strategy is a 'slope-weighted' average of the trading strategies when $\mathbf{q} = \mathbf{0}$ and $\mathbf{q} = \mathbf{1}$. Whether $x_t(r, \eta, \mathbf{0}) > x_t(r, \eta, \mathbf{1})$ depends on what strategic traders know. Substituting for $x_t(r, \eta, \mathbf{0})$ and $x_t(r, \eta, \mathbf{1})$

²⁵ This is shown by noting that if the variance of r is zero, then $x_t(r, \eta, \mathbf{0}) = x_t(r, \eta, \mathbf{1}) = x_t(r, \eta, \mathbf{q})$ and so $\alpha_t(\mathbf{1}) = [N(\delta_t - d_t)/\lambda_{t-1}(\mathbf{q})[1 + N\delta_{t-1}] - \lambda_{t-1}(\mathbf{1})Nd_{t-1}]$.

in Eq. (3),

$$x_t(r, \eta, \mathbf{q}) = \alpha_t(\mathbf{1})[(1 - w_t)(r + \eta - \mu_t^*(\mathbf{0})) + w_t(r + \eta - \mu_t^*(\mathbf{1}))].$$

Previously, we showed that $r + \eta - \mu_t^*(\mathbf{0}) > r + \eta - \mu_t^*(\mathbf{1})$ if and only if $r > E[r|z_{t-1}]$. Hence, if strategic traders have good news about r , they buy shares and $x_t(r, \eta, \mathbf{q}) > x_t(r, \eta, \mathbf{1})$. If they have bad news about r , they sell shares and $x_t(r, \eta, \mathbf{1}) > x_t(r, \eta, \mathbf{q})$. In both cases, strategic traders submit a larger order when there is uncertainty about the time of the release of r . To compare $x_t(r, \eta, \mathbf{q})$ and $x_t(r, \eta, \mathbf{0})$, if $\lambda_t^*(\mathbf{0})/\lambda_t^*(\mathbf{1})$ is small enough, $x_t(r, \eta, \mathbf{0}) > x_t(r, \eta, \mathbf{q})$. If not, then $x_t(r, \eta, \mathbf{q}) > x_t(r, \eta, \mathbf{0})$: the incentive to trade before the information is released dominates the incentive to dissipate their information advantage slowly.

To complete the comparison of the slopes of the equilibrium price functions, using Theorem 1 and our new expression for $x_t(r, \eta, \mathbf{q})$, direct computation shows that $\lambda_t^*(\mathbf{q})$ solves

$$\begin{aligned} \lambda_t(\mathbf{q}) = & \frac{(1 - \alpha_t(\mathbf{1})\lambda_t(\mathbf{q}))\alpha_t(\mathbf{1})}{\lambda_t^*(\mathbf{1})\text{Var}[u_t]} ((1 - w_t)\lambda_t^*(\mathbf{0})\text{Var}[r + \eta|z_{t-1}] \\ & + w_t\lambda_t^*(\mathbf{1})\text{Var}[\eta|z_{t-1}, r]). \end{aligned} \quad (4)$$

The right hand side of that expression is a function of $\lambda_t(\mathbf{q})$ times a ‘slope-weighted’ average of the variance of the sum of the strategic traders’ private information when r is not released and when it is. It is a slope-weighted average because strategic traders’ equilibrium trading strategies are slope-weighted averages of their trading strategies when r is not released and when it is. The term in brackets is larger than $\text{Var}[\eta|z_{t-1}, r]$ and since $\lambda_t^*(\mathbf{0})$ solves

$$\lambda_t(\mathbf{0}) = \alpha_t(\mathbf{0})(1 - \alpha_t(\mathbf{0})\lambda_t(\mathbf{0}))\left(\frac{\text{Var}[r + \eta|z_{t-1}]}{\text{Var}[u_t]}\right)$$

and $\lambda_t^*(\mathbf{1})$ solves

$$\lambda_t(\mathbf{1}) = \alpha_t(\mathbf{1})[1 - \alpha_t(\mathbf{1})\lambda_t(\mathbf{1})]\left(\frac{\text{Var}[\eta|z_{t-1}, r]}{\text{Var}[u_t]}\right),$$

we see that $\lambda_t^*(\mathbf{q}) > \lambda_t^*(\mathbf{1})$. Further, if

$$(1 - w_t)\lambda_t^*(\mathbf{0})\text{Var}[r + \eta|z_{t-1}] + w_t\lambda_t^*(\mathbf{1})\text{Var}[\eta|z_{t-1}, r] < \text{Var}[r + \eta|z_{t-1}]$$

then $\lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{q}) > \lambda_t^*(\mathbf{1})$.²⁶ That is, if the slope-weighted average variance of the strategic traders’ private information is smaller than the variance when r is

²⁶ If price sensitivity were the same when $\mathbf{q} = \mathbf{0}$ and when $\mathbf{q} = \mathbf{1}$, then the relationships between the slopes would follow immediately. Since the strategic traders’ trading strategies are not a weighted average of the trading strategies but a slope-weighted average, the slope of the equilibrium price function is not a weighted average of the slopes when $\mathbf{q} = \mathbf{0}$ and $\mathbf{q} = \mathbf{1}$.

not released, then price is more sensitive to an order imbalance when the vector of release probabilities is identically zero; less sensitive when r may be released during trading; and least sensitive when r is released prior to the start of trading. The slope-weighted average variance will be smaller than the variance when r is not released if the difference between the slopes when $\mathbf{q} = \mathbf{0}$ and $\mathbf{q} = \mathbf{1}$ is not too large. It will also tend to be smaller if the prior variance of r is relatively small, if the prior variance of η is relatively large and/or r and η are not highly correlated. If the slope-weighted average is larger, then $\lambda_t^*(\mathbf{q}) > \lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{1})$.²⁷ Again, the intuition is that if the slope-weighted average is larger, the specialist/dealers' residual variance is greater and so they increase price sensitivity.

2. Implications

2.1. Comparative statics

We begin by showing how strategic traders' orders depend on the probability that r is released in the next period. From Eq. (3), since $x_t(r, \eta, \mathbf{0})$ and $x_t(r, \eta, \mathbf{1})$ are independent of q_{t+1} ,

$$\frac{\partial x_t(r, \eta, \mathbf{q})}{\partial q_{t+1}} = \frac{1}{\lambda_t^*(\mathbf{1})} (\lambda_t^*(\mathbf{1})x_t(r, \eta, \mathbf{1}) - \lambda_t^*(\mathbf{0})x_t(r, \eta, \mathbf{0})) \frac{\partial w_t}{\partial q_{t+1}} \equiv \mathcal{W} \frac{\partial w_t}{\partial q_{t+1}}.$$

Recalling that δ_t and d_t are coefficients in the strategic traders' equilibrium trading strategies, and using expressions developed in Theorem 1,

$$\frac{\partial w_t}{\partial q_{t+1}} = \frac{-N\lambda_t(\mathbf{q})}{(\lambda_t(\mathbf{q}) + N(\delta_t\lambda_t(\mathbf{q}) - d_t\lambda_t^*(\mathbf{1})))^2} \left[\frac{\partial d_t}{\partial q_{t+1}} + N \left(\delta_t \frac{\partial d_t}{\partial q_{t+1}} - d_t \frac{\partial \delta_t}{\partial q_{t+1}} \right) \right].$$

In the proof of Theorem 1, we showed that $d_t, \delta_t > 0$ and direct computation shows that $\partial \delta_t / \partial q_{t+1} > 0$. Unfortunately, $\partial d_t / \partial q_{t+1} = 2\phi_{t+1}(\mathbf{1}) - 2\lambda_t(\mathbf{1})f_{t+1} - \mathcal{F}_{t+1}\lambda_t(\mathbf{q})$.²⁸ The first term times $E[(r + \eta - \mu_{t+1}^*(\mathbf{1}))^2 | \mathbf{z}_{t+1}, r, \eta]$ is a strategic trader's expected profits from trading in all future periods when r is released prior to the start of trading. The last two terms multiply the same expectation and the product is the difference between a strategic trader's expected profits from trading in all future periods if r is not released and the profits if it is released in the future. Thus, $\partial d_t / \partial q_{t+1} > 0$ if a strategic trader's expected future trading profits when r does not happen to be released are at least twice as great as his expected profits if it happens to be released in the next

²⁷ See the proof of Proposition 4.

²⁸ The definitions of f_{t+1} and $\mathcal{F}_{t+1}$ are in the proof of Theorem 1.

period. This condition is unlikely to be met if the prior variance of r is small, prior variance of η is large and/or r and η are not highly correlated. Since we believe that this is the usual case, we expect that $\partial d_t / \partial q_{t+1} < 0$. Hence, if the probability of release in the next period rises and $\mathcal{W} > 0$, then each strategic trader submits a larger order and if $\mathcal{W} < 0$, each submits a smaller order.²⁹ Intuitively, the trading strategy when the release is uncertain is a slope-weighted average of the trading strategies when it is not released during trading and when it is released prior to the start of trading. Therefore, when the probability of release in the next period rises, the slope-weights change. If w_t increases, more weight is placed on the trading strategy used when $q = 1$ and so the amount traded rises if the strategic traders would have traded more when $q = 1$ than when $q = 0$.

We can also determine how a change in the probability that r is released in the next period affects price sensitivity: the sign of $\partial \lambda_t^*(q) / \partial q_{t+1}$. Using Eq. (4)

$$\frac{\partial \lambda_t^*(q)}{\partial q_{t+1}} = \frac{\alpha_t(\mathbf{1})(1 - \alpha_t(\mathbf{1})\lambda_t^*(q))}{\lambda_t^*(\mathbf{1})\text{Var}[u_t]} (\lambda_t^*(\mathbf{1})\text{Var}[\eta|z_{t-1}, r] - \lambda_t^*(\mathbf{0})\text{Var}[r + \eta|z_{t-1}]) \frac{\partial w_t}{\partial q_{t+1}}.$$

If strategic traders trade more when the probability of release in the next period is higher (for any realization of r and η) and if the slope-weighted average variance is smaller than the variance of $r + \eta$, then specialist/dealers respond by reducing price sensitivity.

2.2. Strategic traders' expected profits/exogenous traders' expected losses

Computing strategic traders' profits from trading on their private information is greatly simplified by observing that specialist/dealers earn zero expected profits in each period. Since strategic traders trade only when they expect to earn a profit, their expected profits equal exogenous traders' expected losses. Exogenous traders' expected losses in any period are³⁰

$$E[u_t(v - p_t(z_t; q))|z_{t-1}] = -\lambda_t^*(q)\sigma_u^2 \quad (5)$$

²⁹ If strategic traders have bad news about both r and η , they sell shares, $0 > x_t(r, \eta, \mathbf{1}) > x_t(r, \eta, \mathbf{0})$ and therefore $\mathcal{W} > 0$. If strategic traders have good news about both r and η , they buy shares, $x_t(r, \eta, \mathbf{1}), x_t(r, \eta, \mathbf{0}) > 0$. If, in addition, $x_t(r, \eta, \mathbf{1}) > x_t(r, \eta, \mathbf{0})$ and $\lambda_t^*(\mathbf{0})/\lambda_t^*(\mathbf{1})$ is small enough, $\mathcal{W} > 0$. If they have good news about η but bad news overall (sufficiently bad news about r) then $x_t(r, \eta, \mathbf{1}) > 0$ and $x_t(r, \eta, \mathbf{0}) < 0$ and therefore $\mathcal{W} > 0$. In all of these situations, an increase in the probability of release in the next period causes strategic traders to submit larger current orders.

³⁰ See the proof of Proposition 5.

Exogenous traders' expected losses are the variance of their aggregate trade times the slope of the equilibrium price function. In Section 1, we showed that price sensitivity to current order imbalances depends on how r is released and that $\lambda_t^*(\mathbf{0}), \lambda_t^*(\mathbf{q}) > \lambda_t^*(\mathbf{1})$. Thus, exogenous traders' losses are smallest when r is released prior to the start of trading. In the usual case when the specialist/dealers' slope-weighted average residual variance is smaller than their residual variance when $\mathbf{q} = \mathbf{0}$, then $\lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{q})$ and so exogenous traders' expected losses are largest if r is not released during trading. The intuition is that specialist/dealers are least vulnerable to strategic traders when $\mathbf{q} = \mathbf{1}$, somewhat more vulnerable when r may be released during trading and most vulnerable when $\mathbf{q} = \mathbf{0}$. To avoid losses when they are more vulnerable, they increase price sensitivity. Doing so ensures that they earn zero expected profits but also results in transferring additional wealth from exogenous to strategic traders.

Total expected profits earned by strategic traders are minus the sum of exogenous traders' expected losses. Since each strategic trader knows the same information and uses the same trading strategy, each earns $\frac{1}{N}$ of this total. When there is uncertainty about the time of the release of r , strategic traders' expected profits are

$$\sum_{t=1}^T \left[\left(\prod_{\tau=1}^t (1 - q_\tau) \right) \lambda_t^*(\mathbf{q}) \sigma_u^2 + \left(1 - \prod_{\tau=1}^t (1 - q_\tau) \right) \lambda_t^*(\mathbf{1}) \sigma_u^2 \right].$$

If $\lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{q}) > \lambda_t^*(\mathbf{1})$, strategic traders' profits are largest when r is not released during trading, smaller when there is uncertainty about the time of release and smallest when r is released prior to the start of trading. Further, as shown above, increasing the probability that r is released in the next period reduces price sensitivity and thus reduces strategic traders' expected profits and exogenous traders' expected losses.³¹

Thus, we conclude that if one seeks to minimize exogenous traders' expected losses, thereby minimizing strategic traders' expected profits, r should be released as soon as possible. In prior work, there was no offsetting benefit to releasing r later. In our model, because r and η are (weakly) positively correlated, we can show that there is an offsetting benefit – more private information is acquired and thus available for imbedding in price through trade. To show this, compute the difference between strategic traders' profits in period t when r is not released during trading, $\pi_t(\mathbf{0})$, and their profits when r is released prior to trading

³¹ If one expects leakage of information prior to its release, this leakage must be viewed as additional, currently unmodelled private information for traders. If such additional private information is present, it provides a motivation for specialist/dealers to increase rather than reduce price sensitivity.

in period t , $\pi_t(\mathbf{1}) - \pi_t(\mathbf{0}) - \pi_t(\mathbf{1})$, is proportional to³²

$$\text{Var}[r|z_{t-1}] + 2\text{Cov}[r, \eta|z_{t-1}] + \frac{\text{Cov}[r, \eta|z_{t-1}]}{\text{Var}[r|z_{t-1}]}.$$

Thus, their extra profits in t when r is not released during trading are decreasing in the covariance of r and η . This produces a trade-off between minimizing exogenous traders' losses and maximizing the information imbedded in price. If r and η are independent, then releasing r sooner and reducing exogenous traders' expected losses does not create an offsetting cost because it does not reduce the variance of η . While it does lower strategic traders' profits, it does so by eliminating their profits from knowing r , not their profits from knowing η . Consequently, releasing r as soon as possible imbeds it more rapidly in price without reducing incentives to acquire η . If r and η are independent, minimizing exogenous traders' expected losses by releasing r as rapidly as possible is optimal. Unfortunately, when r and η are positively correlated, releasing r reduces incentives to acquire η and the costs of not having η imbedded in the asset's price may offset the benefit of reducing exogenous traders' losses.

2.3. Information acquisition

In Section 1, the information asymmetry between specialist/dealers and traders was taken as given. In this subsection, we explore investors' incentives to acquire private information, the type(s) of information they choose to acquire and the effect of uncertainty about the time of the release of r . This can be done most clearly when r and η are independent.³³ Whether strategic traders all choose to acquire r , η or both depends on what they expect to earn in each case and how costly information acquisition is. Strategic traders' expected profits when they acquire only η are $\lambda_t^*(\mathbf{1})\sigma_u^2$ in t regardless of how r is released because, if they do not know it, the time of its release is irrelevant. When they acquire only r , they only make profits prior to its release. Once it is released, their information advantage is eliminated and their expected trading profits are zero. Further, once r is released, specialist/dealers are no longer at an information disadvantage and so they set price equal to the *common* expectation of the value of the asset and do not adjust price to the order imbalance because

³² This can be shown by using Eq. (5), substituting for $\lambda_t^*(\mathbf{0})$ and $\lambda_t^*(\mathbf{1})$ to show that $\pi_t(\mathbf{0}) - \pi_t(\mathbf{1})$ is proportional to $\text{Var}[r + \eta|z_{t-1}] - \text{Var}[\eta|z_{t-1}, r]$. Also,

$$\text{Var}[\eta|z_{t-1}, r] = \text{Var}[\eta|z_{t-1}] \left(1 - \frac{\text{Cov}[r, \eta|z_{t-1}]^2}{\text{Var}[r|z_{t-1}]\text{Var}[\eta|z_{t-1}]} \right).$$

Substituting yields the result.

³³ If they are positively correlated, acquiring information about one provides information about the other and clouds the comparison.

it no longer contains information: the slope of the equilibrium price function is zero. Thus, in each period prior to the release of r , the strategic traders' expected profits are $\lambda_t^*(q)\sigma_u^2$ where in the expression for $\lambda_t^*(q)$, one replaces $\text{Var}[r + \eta | z_{t-1}]$ with $\text{Var}[r | z_{t-1}]$ because the strategic traders (by hypothesis) no longer know η .

If the costs of acquiring r and η are approximately the same and the variance of r is less than or equal to the variance of η , then traders prefer to acquire η because in each period, their expected profits are at least as large and they expect to be able to trade in more periods. Further, an early release of r makes traders even more likely to prefer to acquire η . In either case, however, only if the incremental expected trading profits from acquiring r exceed the cost of acquisition will traders also acquire r . Recall that η can be thought of as the outcome of a firm's strategic plan or the outcome of undertaken projects. Thus, its variance is likely to be larger than the variance of information that is regularly scheduled for release such as earnings or dividends. If strategic traders tend to be upper-level management, their cost of acquiring either r or η is likely to be approximately the same. If, however, strategic traders tend to be lower-level management or people outside of the firm (financial analysts), one might believe that the cost of acquiring r is less than the cost of acquiring η . In this case, they prefer to acquire η only if expected profits from doing so more than offset the added costs. Finally, the benefits of acquiring r decline with an increase in the probability of its release. There are two reasons for this: first, traders expect that there is less time to exploit the information advantage and second, expected profits from exploiting the advantage in any period are smaller.

Our results can be summarized as follows. If the cost of acquiring r and η is approximately the same, then:

- If the variance of η exceeds the variance of r and strategic traders acquire only one, they choose to acquire η . (They can trade on it longer and the profits earned in each period are greater.)
- As the correlation between r and η increases, the profits from acquiring η decline and may result in neither being acquired. (If r is highly correlated with η , then once r is released, trading on η is not very profitable.)
- If r and η are positively correlated, then an increase in the probability that r will be released soon reduces strategic traders' profits from trading on η and may result in neither being acquired.

Our results also have implications for the design of public information systems.

- If the information released by a firm's public information systems (r) is uncorrelated with the long-lived information (η), then it is optimal to release the information as rapidly as possible. (The short-lived information is impounded in price more quickly and does not reduce the strategic traders'

incentives to acquire the long-lived information, leading to unambiguously more informative prices.)

- If the information released by a firm's public information systems (r) is positively correlated with the long-lived information (η), the benefits of rapid release must be compared to the costs of inhibiting the acquisition of long-lived information. (Rapid release will incorporate the short-lived information quickly but may inhibit the incorporation of long-lived information into the price by reducing or eliminating the strategic traders' incentives to acquire it.)

Our information acquisition results can be contrasted with those in rational expectations models. Grundy and McNichols (1989) focus on multiperiod rational expectations equilibria in which additional trades allow traders to infer other traders' private information. In such equilibria, they show that a public release may partially substitute for additional trading. As a result, public release reduces the benefits of acquiring private information. In rational expectations models such as those studied by Holden and Subrahmanyam (1996) and Lundholm (1991), information acquisition depends on traders' risk tolerance and the precision of publicly released information. If traders are risk averse, they tend to privately acquire information that will be publicly released in order to reduce the amount of risk associated with information acquisition. Alternatively, if traders can choose to acquire neither, one or two signals, their decision depends on (1) whether the signals are complements, (2) whether there is a correlated publicly released signal and (3) the amount of noise in the signals.

2.4. *Special types of announcements*

2.4.1. *Announcements of forthcoming information releases*

Uncertainty in the time of the release of r allows us to study the effects of an announcement that there will be a public release of information in the near future. For example, firms frequently call press conferences or schedule meetings with analysts or announce that they will release earnings on a particular day. Such announcements increase the probability of public release and, in Section 2.1, we showed that an increase in the probability of release of r in the next trading period induces strategic traders to submit a larger order for any realization of r and η and specialist/dealers to make price less sensitive to the current order imbalance. Combined, these results allow us to conclude that announcing that there may be another announcement in the near future reduces price sensitivity, strategic traders' expected profits and exogenous traders' expected losses.

2.4.2. *Anticipated versus unanticipated announcements*

We define an anticipated announcement (e.g., regularly scheduled earnings and dividend announcements) as one for which it is believed that information

will not be released until just prior to trading in τ while an unanticipated announcement (e.g., unexpected events such as the sudden death of a CEO, the filing of a lawsuit or the first rumor of a merger/takeover) is one for which it is believed that there will be no announcement during trading. Thus, for anticipated announcements $\mathbf{q}_1 = (0, 0, \dots, 0, 1, 1, \dots, 1)$ where the switch from 0 to 1 occurs in the τ th position while for unanticipated announcements $\mathbf{q} = \mathbf{0}$. The differences are captured by comparing $\lambda_t^*(\mathbf{q}_1)$ and $\lambda_t^*(\mathbf{0})$ keeping in mind that after the unexpected release, price sensitivity is the same. Thus, all differences between anticipated and unanticipated announcements arise prior to the release.

We find that price sensitivity is smaller for anticipated than for unanticipated announcements and, for given realizations of r and η , strategic traders place a larger order if the announcement is anticipated. The intuition for this result can be obtained by considering how price sensitivity changes when the vector of probabilities of release changes from $\mathbf{0}$ to $\mathbf{q}_1$. Such a change is like increasing the probability of release in period τ from 0 to 1. Previously, we showed that if the probability of release in the next period increases, price sensitivity declines and strategic traders increase their order for any given realizations of r and η . The reason for this is that they are more likely to lose part of their information advantage in the next period and so the future cost of a larger order is smaller. Specialist/dealers respond by making price less sensitive because larger order imbalances no longer indicate as great a difference between their expectation of $r + \eta$ and the realization of $r + \eta$ but instead simply indicate more aggressive trading. This reduction in price sensitivity then folds back to period $\tau - 2$ because the slope of the equilibrium price function is smaller in $\tau - 1$ and so the cost of a larger order placed in $\tau - 2$ declines, inducing strategic traders to trade more aggressively. This increase in trading aggression leads specialist/dealers to reduce price sensitivity in $\tau - 2$. Repeating this argument, we see that relative to price sensitivities when $\mathbf{q} = \mathbf{0}$, price sensitivities when $\mathbf{q} = \mathbf{q}_1$ are all smaller. As a result, trading profits when the announcement is anticipated are smaller than when the announcement is unanticipated.

In summary,

- An anticipated announcement forces strategic traders to hurry to exploit their short-lived private information. Thus, they trade more aggressively, prices are more informative and specialist/dealers make the effective bid/ask spread smaller.

3. Conclusions

This paper examines how public information releases affect investors' decisions to acquire and trade on private information about the value of a firm. In

a model of a stock exchange, we introduce two types of private information: one that may be publicly released in any trading period, r , and one that is not publicly released during trading, η . The probability that r is released in any period is exogenous, allowing us to specify the probabilities in different ways to represent different types of information releases. For example, r can be thought of as private information about a regularly scheduled earnings or dividend announcement, or an unusual, significant event that must be reported in an 8-K filing such stock splits, stock repurchases, mergers and acquisitions, or changes in upper management. η can be thought of as private information about events whose outcomes are normally revealed through the passage of time such as strategic plans or long-term projects.

Strategic traders, those investors with these two types of private information, can trade repeatedly thereby forming trading strategies. Since r is probabilistically released in any given period, the strategic traders are uncertain about the number of trades they can make prior to losing part of their information advantage. We study linear equilibria in which prior order imbalances and any publicly released information are captured by the intercept of the specialist/dealers' equilibrium price function and the current order imbalance is captured by the slope of this price function. The strategic traders correctly infer the specialist/dealers' price functions and balance profits from additional trades in a period with the induced change in the price at which they may trade both in the current and all future periods.

To investigate incentives to acquire private information, we compute strategic traders' expected profits and show that expected trading profits from acquiring η should, in general, exceed expected trading profits from acquiring r . We also show that expected trading profits decline in the correlation between r and η because the higher the correlation, the smaller the strategic traders' information advantage after r is released. Thus, we conclude that expected trading profits from knowing η depend on when r is released: private information acquisition depends on characteristics of their joint distribution as well as uncertainty in the time of release.

Our analysis has a number of implications, some of which are testable. To draw such implications, we rely on the fact that the information that r and η represent, the degree to which they are correlated and their prior variances are 'determined' by the firm's public information systems. If the systems ensure that r and η are uncorrelated, then our analysis provides strong support for requiring that r be released as rapidly as possible. Doing so reduces exogenous traders' losses by equalizing information access and impounds r in price more quickly without reducing private incentives to learn η . However, if the systems ensure that r and η are positively correlated, then our analysis suggests that rapid release *reduces* private incentives to learn η and may eliminate them. Hence, fewer traders acquire η and so it is impounded in price more slowly, if at all. This raises the possibility of misdirected investment, and such costs need to be

balanced with the benefits of rapidly releasing r . However, the degree to which r and η are correlated and the associated impact on the incentives to privately acquire η are empirical matters that await additional research.

Our analysis suggests that the level of private information trading depends on both the nature of the firm itself and its public information system. For example, the content of current public information releases (e.g., earnings releases) of risky, high-tech, start-up firms are likely to be relatively uncorrelated with their long-term prospects. On the other hand, the content of current public information releases of well-established firms in stable, mature industries is likely to be highly correlated with their long-term prospects. Our analysis predicts more private information acquisition and trading for the former type of firm. Our analysis also suggests that, in general, there is less private information acquisition and trading associated with routine information releases such as earnings and dividends announcements and more associated with the results of long-term projects or strategic plans.

There are a number of potential extensions of the basic model. First, one could introduce heterogeneously informed traders. Doing so allows for differences in interpretation of publicly released information and permits a deeper understanding of the interaction of private information acquisition and public releases. Second, an extension of the analysis to other models of stock exchanges will help determine the extent to which our results depend on the particular choice of model for a stock exchange. Third, managers have *some* flexibility in choosing what information to publicly release and when to release it. Including their motives is an important next step in the analysis of the interaction of public information release and asset pricing.

Acknowledgements

We wish to thank the Editor, Avanidhar Subrahmanyam, as well as Bob Forsythe, Craig Holden, Renee Price, Bal Radhakrishnan, Jerry Salamon, Tom Stober, Gerry Suchanek and participants at the 1995 American Accounting Association meetings, the 1995 Western Finance Association meetings and the Indiana University Accounting workshop for their comments. Watts gratefully acknowledges Indiana University research support. As always, we are responsible for any errors.

Appendix A.

Proposition 1. For all N , if r is not released during trading, in equilibrium, specialist/dealers all choose the following price functions:

$$p_t(z_t, \mathbf{0}) = \mu_t^*(\mathbf{0}) + \lambda_t^*(\mathbf{0})z_t \quad t = 1, 2, \dots, T$$

where $\mu_t^*(\mathbf{0}) = E[v|z_{t-1}, \mathbf{0}]$, $\lambda_t^*(\mathbf{0})$ is the solution to

$$\lambda_t(\mathbf{0}) = \left(\frac{NA_t(\mathbf{0})\alpha_t(\mathbf{0})}{\text{Var}[u_t]} \right) \text{Var}[r + \eta|z_{t-1}]$$

and strategic traders all choose the same trading strategies:

$$x_t(r, \eta, \mathbf{0}) = \frac{\alpha_t(\mathbf{0})}{N}(r + \eta - \mu_t^*(\mathbf{0})), \quad t = 1, 2, \dots, T.$$

Proof. The proof is by induction. In the text, $\mu_T^*(\mathbf{0})$, $\lambda_T^*(\mathbf{0})$, $x_T(r, \eta, \mathbf{0})$ and $V_T(\mathbf{0})$ were computed as were their values at $T-1$. Thus, we must show that if the equations in Proposition 1 are true for periods $T, T-1, \dots, t+1$ then they are true for period t . We compute $V_{t+1}(\mathbf{0})$ by substituting $x_{t+1}(r, \eta, \mathbf{0})$ into a trader's $t+1$ period objective function to obtain $(\gamma_{t+1}(\mathbf{0}) + \phi_{t+1}(\mathbf{0})(1 - \alpha_{t+1}(\mathbf{0})\lambda_{t+1}(\mathbf{0}))^2)E[r + \eta - \mu_{t+1}(\mathbf{0})|z_t, r, \eta, \mathbf{0}]$. The i th strategic trader's objective function becomes

$$\begin{aligned} & \left(r + \eta - \mu_t(\mathbf{0}) - \lambda_t(\mathbf{0}) \sum_{j=1}^N x_{j,t} \right) x_{i,t} + \phi_i(\mathbf{0}) \left[(r + \eta - \mu_t(\mathbf{0}))^2 \right. \\ & \quad \left. - 2(r + \eta - \mu_t(\mathbf{0}))\lambda_t(\mathbf{0}) \sum_{j=1}^N x_{j,t} + \lambda_t(\mathbf{0})^2 \left(\left(\sum_{j=1}^N x_{j,t} \right)^2 + \text{Var}[u_t] \right) \right]. \end{aligned}$$

Maximizing with respect to $x_{i,t}$ and solving in the usual way shows that, in equilibrium, all strategic traders use the same trading strategy $x_t(r, \eta, \mathbf{0}) = \alpha_t(\mathbf{0})(r + \eta - \mu_t(\mathbf{0}))/N$, where $\alpha_t(\mathbf{0})$ and $A_t(\mathbf{0})$ (used in the definition of $\alpha_t(\mathbf{0})$) are as defined in the text.

Turning to the equilibrium price function, $p_t(z_t; \mathbf{0}) = E[v|z_t, \mathbf{0}] = \mu_t(\mathbf{0}) + \lambda_t(\mathbf{0})z_t$. Taking expectations with respect to z_t conditional on z_{t-1} yields $E[v|z_{t-1}, \mathbf{0}] = \mu_t(\mathbf{0}) + \lambda_t(\mathbf{0})E[z_t|z_{t-1}, \mathbf{0}]$. Since $E[z_t|z_{t-1}, \mathbf{0}] = E[Nx_t + u_t|z_{t-1}, \mathbf{0}]$, u_t is independent of all random variables and $E[u_t] = 0$, after substituting the equilibrium trading strategies, $E[z_t|z_{t-1}, \mathbf{0}] = \alpha_t(\mathbf{0})E[r + \eta - \mu_t(\mathbf{0})|z_{t-1}, \mathbf{0}]$. Since ε is independent of all random variables and $E[\varepsilon] = 0$, $E[v|z_{t-1}, \mathbf{0}] = E[r + \eta|z_{t-1}, \mathbf{0}]$ and so we have $E[v|z_{t-1}, \mathbf{0}] = \mu_t(\mathbf{0}) + \lambda_t(\mathbf{0})\alpha_t(\mathbf{0})(E[v|z_{t-1}, \mathbf{0}] - \mu_t(\mathbf{0}))$, or

$$\mu_t^*(\mathbf{0}) = E[v|z_{t-1}, \mathbf{0}].$$

To compute $\lambda_t^*(\mathbf{0})$, again, start with the equilibrium price function but this time multiply through by z_t and then take expectations with respect to z_t conditional on z_{t-1} to get

$$E[vz_t|z_{t-1}, \mathbf{0}] = \mu_t^*(\mathbf{0})E[z_t|z_{t-1}, \mathbf{0}] + \lambda_t(\mathbf{0})E[z_t^2|z_{t-1}, \mathbf{0}].$$

Since $\mu_t^*(\mathbf{0}) = E[v|z_{t-1}, \mathbf{0}] = E[r + \eta|z_{t-1}, \mathbf{0}]$, we have that $E[z_t|z_{t-1}, \mathbf{0}] = 0$ and

$$\lambda_t(\mathbf{0}) = \frac{\text{Cov}[v, z_t|z_{t-1}, \mathbf{0}]}{\text{Var}[z_t|z_{t-1}, \mathbf{0}]}.$$

Computing the covariance and variance terms and rearranging yields

$$\lambda_t(\mathbf{0}) = NA_t(\mathbf{0})\alpha_t(\mathbf{0})\left(\frac{\text{Var}[r + \eta|z_{t-1}]}{\text{Var}[u_t]}\right).$$

Thus, if $\mu_\tau^*(\mathbf{0})$, $\lambda_\tau^*(\mathbf{0})$, $x_\tau(r, \eta, \mathbf{0})$ and $V_\tau(\mathbf{0})$ are as described in the proposition for $\tau = T, T-1, \dots, t+1$, then we have shown that they are as described in the proposition for $\tau = t$. ■

Corollary 1. If $N \geq 2$, then $\lambda_t^*(\mathbf{0}) > \lambda_{t+1}^*(\mathbf{0})$ for $t = 1, 2, \dots, T-1$.

Proof. Recall that u_t are iid normal random variables. From Proposition 1,

$$\lambda_t(\mathbf{0}) = NA_t(\mathbf{0})\alpha_t(\mathbf{0})\left(\frac{\text{Var}[r + \eta|z_{t-1}]}{\sigma_u^2}\right).$$

Since $r + \eta$ is normally distributed,

$$\begin{aligned}\text{Var}[r + \eta|z_{t-1}] &= \text{Var}[r + \eta|z_{t-2}] \left(1 - \frac{\text{Cov}[r + \eta, z_{t-1}|z_{t-2}]^2}{\text{Var}[r + \eta|z_{t-2}]\text{Var}[z_{t-1}|z_{t-2}]}\right) \\ &\equiv \text{Var}[r + \eta|z_{t-2}](1 - \lambda_{t-1}(\mathbf{0})\alpha_{t-1}(\mathbf{0})).\end{aligned}$$

Thus,

$$\lambda_{t+1}(\mathbf{0}) = NA_{t+1}(\mathbf{0})\alpha_{t+1}(\mathbf{0})(1 - \lambda_t(\mathbf{0})\alpha_t(\mathbf{0}))\left(\frac{\text{Var}[r + \eta|z_{t-1}]}{\sigma_u^2}\right).$$

Using the equation for $\lambda_t^*(\mathbf{0})$ and noting that $(1 - \lambda_t(\mathbf{0})\alpha_t(\mathbf{0})) = 1/(1 + NA_t(\mathbf{0}))$, we can substitute and compute

$$\frac{(\lambda_t(\mathbf{0}) - \lambda_{t+1}(\mathbf{0}))\sigma_u^2}{\text{Var}[r + \eta|z_{t-1}]} = A_t(\mathbf{0})\alpha_t(\mathbf{0}) - A_{t+1}(\mathbf{0})\alpha_{t+1}(\mathbf{0})\left(\frac{1}{1 + NA_t(\mathbf{0})}\right).$$

Simplifying, the sign of the lhs is the sign of

$$\lambda_{t+1}(\mathbf{0})A_t(\mathbf{0})^2(1 + NA_{t+1}(\mathbf{0})) - \lambda_t(\mathbf{0})NA_{t+1}(\mathbf{0})^2.$$

If $\lambda_{t+1}(\mathbf{0}) \geq \lambda_t(\mathbf{0})$ then $\lambda_{t+1}(\mathbf{0})A_t(\mathbf{0})^2(1 + NA_{t+1}(\mathbf{0})) - \lambda_t(\mathbf{0})NA_{t+1}(\mathbf{0})^2 \leq 0$. Rearranging,

$$(1 + NA_{t+1}(\mathbf{0}))A_t(\mathbf{0}) \leq A_{t+1}(\mathbf{0})^2\left(\frac{\lambda_t(\mathbf{0})}{\lambda_{t+1}(\mathbf{0})}\right).$$

which holds if and only if $(1 + NA_{t+1}(\mathbf{0}))A_t(\mathbf{0})^2 \leq A_{t+1}(\mathbf{0})^2$. Substituting for $A_t(\mathbf{0})$ and rearranging

$$\left[(1 + NA_{t+1}(\mathbf{0}))^2 - 2\lambda_t(\mathbf{0}) \left(\frac{A_{t+1}(\mathbf{0})}{\lambda_{t+1}(\mathbf{0})} + \phi_{t+1}(\mathbf{0}) \right) \right]^2 \leq A_{t+1}(\mathbf{0})^2 (1 + NA_{t+1}(\mathbf{0}))^3.$$

Since we have supposed that $\lambda_t(\mathbf{0}) \leq \lambda_{t+1}(\mathbf{0})$, we can substitute for $\lambda_t(\mathbf{0})$ making the lhs smaller. Substituting for $\phi_{t+1}(\mathbf{0})$ and rearranging yields

$$[(1 + NA_{t+1}(\mathbf{0}))^2 - (1 + A_{t+1}(\mathbf{0}))]^2 \leq (1 + NA_{t+1}(\mathbf{0}))^3 A_{t+1}(\mathbf{0})^2.$$

The lhs is larger than $[(1 + NA_{t+1}(\mathbf{0}))^2 - (1 + NA_{t+1}(\mathbf{0}))]$ and so $\lambda_t(\mathbf{0}) \leq \lambda_{t+1}(\mathbf{0})$ implies $N^2 < (1 + NA_{t+1}(\mathbf{0}))$. But this is false if $N \geq 2$ since $A_{t+1}(\mathbf{0}) \leq 1$ and so $\lambda_t(\mathbf{0}) > \lambda_{t+1}(\mathbf{0})$. ■

Proposition 2. For all N , if public information r is released prior to the start of trading, in equilibrium, specialist/dealers all choose the following price functions:

$$p_t(z_t, r, \mathbf{1}) = \mu_t^*(\mathbf{1}) + \lambda(\mathbf{1})_t^* z_t, \quad t = 1, 2, \dots, T$$

where $\mu_t^*(\mathbf{1}) = E[v|z_{t-1}, r]$, $\lambda_t^*(\mathbf{1})$ is the solution to

$$\lambda_t(\mathbf{1}) = \left(\frac{NA_t(\mathbf{1})\alpha_t(\mathbf{1})}{\text{Var}[u_t]} \right) \text{Var}[\eta|z_{t-1}, r]$$

and strategic traders all choose the same trading strategies:

$$x_t(r, \eta, \mathbf{1}) = \frac{\alpha_t(\mathbf{1})}{N} (r + \eta - \mu_t^*(\mathbf{1})), \quad t = 1, 2, \dots, T.$$

Proof. The proof mimics the proof of Proposition 2 and is therefore omitted. ■

Corollary 2. If $N \geq 2$, then $\lambda_t^(\mathbf{1}) > \lambda_{t+1}^*(\mathbf{1})$ for $t = 1, 2, \dots, T - 1$.*

Proof. The proof mimics the proof of Corollary 1 and is therefore omitted. ■

Proposition 3

$$\frac{\lambda_t^*(\mathbf{0})}{\lambda_t^*(\mathbf{1})} = \sqrt{\frac{\text{Var}[r + \eta|z_{t-1}]}{\text{Var}[\eta|z_{t-1}, r]}}$$

and so $\lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{1})$.

Proof. We begin by computing $\lambda_{t+1}^*(0)/\lambda_t^*(0)$ using equations developed in the proof of Proposition 1,

$$\lambda_t^*(0) = \alpha_t(0) \left(\frac{\text{Var}[r + \eta | \mathbf{z}_{t+1}]}{\sigma_u^2} \right)$$

and $\text{Var}[r + \eta | \mathbf{z}_{t+1}] = \text{Var}[r + \eta | \mathbf{z}_t](1 - \alpha_t(0)\lambda_t^*(0))$. Hence

$$\left(\frac{\lambda_{t+1}^*(0)}{\lambda_t^*(0)} \right)^2 = \frac{A_{t+1}(0)}{A_t(0)} \left(\frac{1 + NA_t(0)}{1 + NA_{t+1}(0)} \right).$$

Next, recall that $\phi_t(0) = [\gamma_{t+1}(0) + \phi_{t+1}(0)(1 - \alpha_{t+1}(0)\lambda_{t+1}^*(0))^2]$. Substituting and rearranging,

$$\frac{\lambda_{t+1}^*(0)}{\lambda_t^*(0)} = \frac{(1 + A_{t+1}(0))}{1 - A_{t+1}(0)} \left(\frac{1}{1 + NA_{t+1}(0)} \right)^2, \quad t < T - 1.$$

Squaring this, setting it equal to the previous expression and rearranging,

$$(1 + A_{t+1}(0))^2 A_t(0) = A_{t+1}(0)(1 + NA_t(0))(1 + NA_{t+1}(0))^2(1 - A_{t+1}(0)).$$

Following the same procedures using the analogous expressions when r is released prior to the start of trading produces exactly the same equation and so $A_t(0) = A_t(1) \equiv A_t$. Given this,

$$\lambda_t(0) = NA_t \alpha_t(0) \left(\frac{\text{Var}[r + \eta | \mathbf{z}_t]}{\sigma_u^2} \right)$$

and

$$\lambda_t(1) = NA_t \alpha_t(1) \left(\frac{\text{Var}[\eta | \mathbf{z}_t, r]}{\sigma_u^2} \right).$$

Substituting for $\alpha_t(0)$ and $\alpha_t(1)$

$$\frac{\lambda_t^*(0)}{\lambda_t^*(1)} = \sqrt{\frac{\text{Var}[r + \eta | \mathbf{z}_t]}{\text{Var}[\eta | \mathbf{z}_t, r]}}.$$

Since the numerator of the rhs exceeds the denominator, $\lambda_t^*(0) > \lambda_t^*(1)$. ■

Theorem 1. For all N , if $q_t \in [0, 1]$ in each period t , in equilibrium, specialist/dealers all choose the following price functions:

$$p_t(\mathbf{z}_t; \mathbf{q}) = \mu_t^*(\mathbf{q}) + \lambda_t^*(\mathbf{q})z_t, \quad t = 1, 2, \dots, T$$

where

$$\mu_t^*(\mathbf{q}) = \begin{cases} E[v | \mathbf{z}_{t-1}, r] \equiv \mu_t^*(1) & \text{if } r \text{ is released prior to } t - 1 \\ E[v | \mathbf{z}_{t-1}] \equiv \mu_t^*(0) & \text{if not} \end{cases}$$

and $\lambda_t^*(q) = \lambda_t^*(\mathbf{1})$ if r was released prior to t and solves

$$\lambda_t(q) = \frac{E[v, z_t | z_{t-1}]}{E[z_t^2 | z_{t-1}]}$$

if not. Strategic traders all choose the same trading strategies:

$$x_t(r, \eta, q) = \begin{cases} x_t(r, \eta, \mathbf{1}) & \text{if } r \text{ was released prior to } t, \\ \frac{\delta_t(r + \eta - \mu_t^*(q)) - d_{t-1}(r + \eta - \mu_t^*(\mathbf{1}))}{\lambda_t^*(q)[1 + N\delta_t] - \lambda_t^*(\mathbf{1})Nd_t} & \text{if not.} \end{cases}$$

Proof. Let

$$A_t \equiv \frac{N\delta_t}{\lambda_t(q)(1 + N\delta_t) - \lambda_t^*(\mathbf{1})Nd_t}, \quad D_t \equiv \frac{Nd_t}{\lambda_t(q)(1 + N\delta_t) - \lambda_t^*(\mathbf{1})Nd_t}$$

with $A_T = D_T = 1$; $\delta_t \equiv 1 - 2(1 - q_{t+1})F_{t+1}\lambda_t(q) - (1 - q_{t+1})\mathcal{F}_{t+1}\lambda_t(\mathbf{1})$; $d_t \equiv 2C_t\lambda_t(\mathbf{1}) + (1 - q_{t+1})\mathcal{F}_{t+1}\lambda_t(q)$; $C_t \equiv q_{t+1}\phi_{t+1}(\mathbf{1}) + (1 - q_{t+1})f_{t+1}$; $F_t \equiv (1 - \lambda_t(q)A_t)A_t/N + (1 - q_{t+1})F_{t+1}(1 - \lambda_t(q)A_t)^2 + C_t(\lambda_t(\mathbf{1})A_t)^2 + (1 - q_{t+1})\mathcal{F}_{t+1}\lambda_t(\mathbf{1})A_t(1 - \lambda_t(q)A_t)$ with $F_T = 0$; $f_t \equiv -\lambda_t(q)D_t^2/N + (1 - q_{t+1})\mathcal{F}_{t+1}(\lambda_t(q)D_t)^2 + C_t(1 - \lambda_t(\mathbf{1})D_t)^2 + (1 - q_{t+1})\mathcal{F}_{t+1}\lambda_t(q)D_t(1 - \lambda_t(\mathbf{1})D_t)$ with $f_T = 0$; $\mathcal{F}_t \equiv D_t(2\lambda_t(\mathbf{1})A_t - 1)/N + 2(1 - q_{t+1})F_{t+1}(1 - \lambda_t(q)A_t)\lambda_t(q)D_t + 2C_t(1 - \lambda_t(\mathbf{1})D_t)\lambda_t(\mathbf{1})A_t + (1 - q_{t+1})\mathcal{F}_{t+1}[(1 - \lambda_t(q)A_t)(1 - \lambda_t(\mathbf{1})D_t) + \lambda_t(q)D_t\lambda_t(\mathbf{1})A_t]$ with $\mathcal{F}_T = 0$. Also, $\gamma_t(\mathbf{0})$, $\gamma_T(\mathbf{1})$, $\phi_t(\mathbf{1})$ are as defined in Section 1.1.2 and 1.2.2 respectively.

In the text, we presented the analysis for periods T and $T - 1$. To complete the induction proof, we assume that the expressions in the Theorem are true for all periods after period t and show that they hold for period t . Note that as soon as r is released, the analysis in Section 1.2 applies so all of the following calculations are done when r is not yet released. Given the expressions, the i th strategic trader chooses $x_{i,t}$ to maximize

$$\begin{aligned} & \left(r + \eta - \lambda_t(q) \sum_{j=1}^N x_{j,t} \right) x_{i,t} + (1 - q_{t+1}) \left(F_{t+1} \left[(r + \eta - \mu_t(q))^2 \right. \right. \\ & \quad \left. \left. - 2\lambda_t(q)(r + \eta) \sum_{j=1}^N x_{j,t} + \lambda_t(q)^2 \left(\left(\sum_{j=1}^N x_{j,t} \right)^2 + \text{Var}[u_t] \right) \right] \right. \\ & \quad \left. + f_{t+1} \left[(r + \eta - \mu_t^*(\mathbf{1}))^2 - 2\lambda_t(\mathbf{1})(r + \eta - \mu_t^*(\mathbf{1})) \sum_{j=1}^N x_{j,t} \right. \right. \\ & \quad \left. \left. + \lambda_t(\mathbf{1})^2 \left(\left(\sum_{j=1}^N x_{j,t} \right)^2 + \text{Var}[u_t] \right) \right] \right) + (1 - q_{t+1})\mathcal{F}_{t+1} \\ & \quad \times \left[(r + \eta - \mu_t(q))(r + \eta - \mu_t^*(\mathbf{1})) - (\lambda_t(\mathbf{1})(r + \eta - \mu_t(q)) \right. \end{aligned}$$

$$\begin{aligned}
& + \lambda_t(\mathbf{q})(r + \eta - \mu_t^*(\mathbf{1})) \sum_{j=1}^N x_{j,t} + \lambda_t(\mathbf{q}) \lambda_t(\mathbf{1}) \left(\left(\sum_{j=1}^N x_{j,t} \right)^2 \right. \\
& \left. + \text{Var}[u_t] \right) \Bigg] + q_{t+1} \phi_{t+1}(\mathbf{1}) \left[(r + \eta - \mu_t^*(\mathbf{1}))^2 \right. \\
& \left. - 2 \lambda_t(\mathbf{q})(r + \eta - \mu_t^*(\mathbf{1})) \sum_{j=1}^N x_{j,t} + \lambda_t(\mathbf{1})^2 \left(\left(\sum_{j=1}^N x_{j,t} \right)^2 + \text{Var}[u_t] \right) \right]
\end{aligned}$$

plus a constant. Computing the first order condition and then solving in the usual way yields

$$x_t(r, \eta, \mathbf{q}) = \frac{A_t}{N}(r + \eta - \mu_t^*(\mathbf{0})) - \frac{D_t}{N}(r + \eta - \mu_t^*(\mathbf{1})).$$

Following the procedures described in the proof of Proposition 1 yields the intercept and the equation that the equilibrium slope must satisfy. ■

Proposition 4. $\lambda_t^*(\mathbf{q}) > \lambda_t^*(\mathbf{1})$. Also, if

$$[(1 - w_t)\lambda_t^*(\mathbf{0})\text{Var}[r + \eta|\mathbf{z}_{t-1}] + w_t\lambda_t^*(\mathbf{1})\text{Var}[\eta|\mathbf{z}_{t-1}, r]] < \text{Var}[r + \eta|\mathbf{z}_{t-1}],$$

then $\lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{q})$.

Proof. By Proposition 3, $\lambda_t^*(\mathbf{0}) > \lambda_t^*(\mathbf{1})$ and so

$$\frac{\lambda_t(\mathbf{q})}{1 - \alpha_t(\mathbf{1})\lambda_t(\mathbf{q})} > \alpha_t(\mathbf{1}) \left(\frac{\text{Var}[\eta|\mathbf{z}_{t-1}, r]}{\text{Var}[u_t]} \right) = \frac{\lambda_t^*(\mathbf{1})}{1 - \alpha_t(\mathbf{1})\lambda_t^*(\mathbf{1})}.$$

Rearranging shows that $\lambda_t^*(\mathbf{q}) > \lambda_t^*(\mathbf{1})$. To show the second inequality, rewrite so that $\lambda_t^*(\mathbf{q})$ solves

$$\lambda_t(\mathbf{q}) = \frac{\alpha_t(\mathbf{0})\lambda_t^*(\mathbf{0})(1 - \alpha_t(\mathbf{1})\lambda_t(\mathbf{q}))}{\lambda_t^*(\mathbf{1})\text{Var}[u_t]} [(1 - w_t)\lambda_t^*(\mathbf{0})\text{Var}[r + \eta|\mathbf{z}_{t-1}] + w_t\lambda_t^*(\mathbf{1})\text{Var}[\eta|\mathbf{z}_{t-1}, r]].$$

Using the condition in the Proposition,

$$\frac{\lambda_t(\mathbf{q})}{1 - \alpha_t(\mathbf{1})\lambda_t(\mathbf{q})} < \alpha_t(\mathbf{0}) \left(\frac{\text{Var}[r + \eta|\mathbf{z}_{t-1}]}{\text{Var}[u_t]} \right) = \frac{\lambda_t^*(\mathbf{0})}{1 - \alpha_t(\mathbf{0})\lambda_t^*(\mathbf{0})}.$$

Rearranging shows that $\lambda_t^*(\mathbf{q}) < \lambda_t^*(\mathbf{0})$. ■

Proposition 5. In t , exogenous traders' expected losses are $E[u_t(v - p_t(\mathbf{z}_t; \mathbf{q}))|\mathbf{z}_{t-1}] = -\lambda_t(\mathbf{q})\sigma_u^2$.

Proof. Substitute for p_t and use $z_t = Nx_t(r, \eta; \mathbf{q}) + u_t$ to obtain

$$E[u_t(v - p_t(z_t; \mathbf{q}))|z_{t-1}] = E[u_t(v - \mu_t^*(\mathbf{q}) - \lambda_t^*(\mathbf{q})(Nx_t(r, \eta; \mathbf{q}) + u_t))|z_{t-1}].$$

Next, observe that $E[x_t(r, \eta; \mathbf{q})|z_{t-1}] = 0$ and using the assumption that u_t is independent of all of the other random variables,

$$E[u_t(v - p_t(z_t; \mathbf{q}))|z_{t-1}] = -\lambda_t^*(\mathbf{q})E[u_t^2].$$

Since $E[u_t] = 0$ and $\text{Var}[u_t] = \sigma_u^2$, we have $E[u_t(v - p_t(z_t; \mathbf{q}))|z_{t-1}] = -\lambda_t^*(\mathbf{q})\sigma_u^2$. ■

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